Institut pour la **Maîtrise des Risques**
Sûreté de Fonctionnement - Management - Cindyniques

AK-SYS-T: Application of AK-SYS method for time-dependent reliability analysis

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Biograpghy

Educations:

- PhD: Material, reliability and structural engineering
- MSc: Industrial and Systems Engineering
- MSc: Mechanical Engineering (Hydro-mechanics)
- BSc: Marine Engineering and Naval Architecture

Work Experience:

- Researcher in structural reliability and maintenance (since 2017)
- Marine engineer (1 year)
- Physics lab instructor and teaching assistant (3 years)
- Freelance interpreter (6 month)

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- Introduction to my Thesis
- Motivation and objectives
- Time-variant reliability
- System reliability: AK-SYS
- Proposed methodology and validation

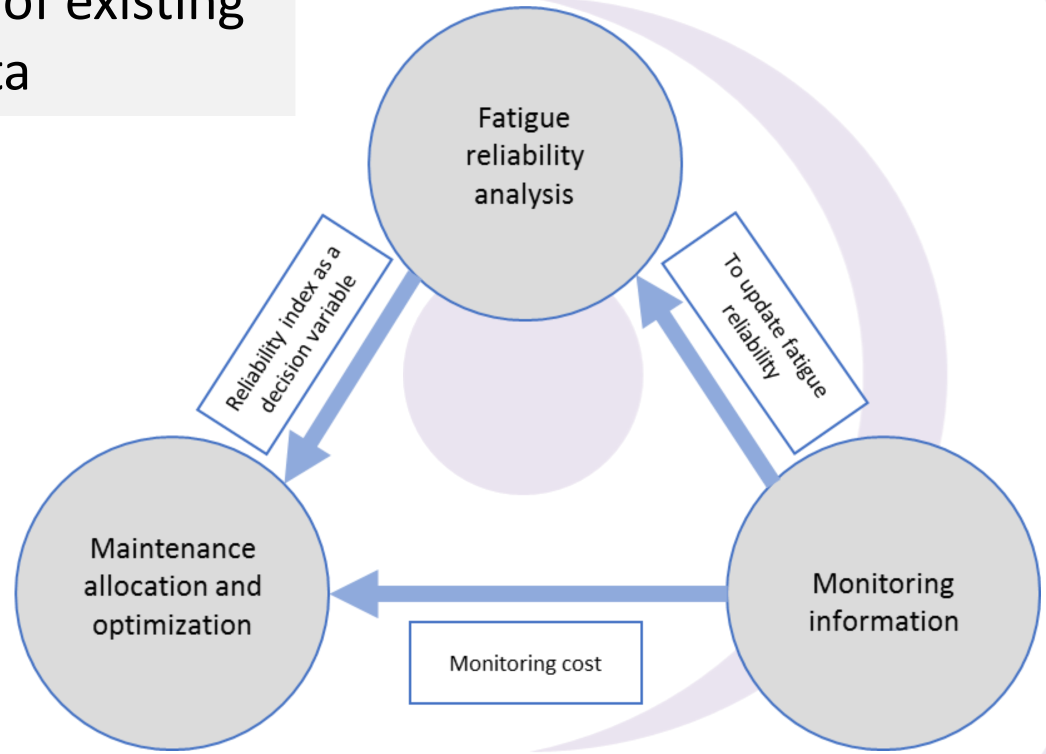
Introduction

► **Thesis topic**: Optimal maintenance planning of existing structures using monitoring data

Step 1: Fatigue reliability analysis
- New time-variant reliability method

Step 2: Employing monitoring data

Step 3: Maintenance optimization



Motivation and objectives

► **Objective**:

To develop an efficient method for time-variant reliability (TVR)

- The indicator to take actions is reliability index
- Fatigue is a time-variant process

► **Challenge**:

A reasonable trade off between accuracy and efficiency for:

- Problems with high-dimension
- Problems with low failure probability

Available methods:

► Out-crossing rate: Rice and PHI2

► Extreme value methods: meta-modeling (SILK, mixed-EGO, …)

(Wang and Wang 2012)
(Hu and Mahdevan 2016)
(Rice 1944)
(Andrieu Renaud et al, 2004)

Time-dependent reliability analysis

Time-dependent problem

▶ Performance function: $G(\boldsymbol{X},\boldsymbol{Y}(t),t)$

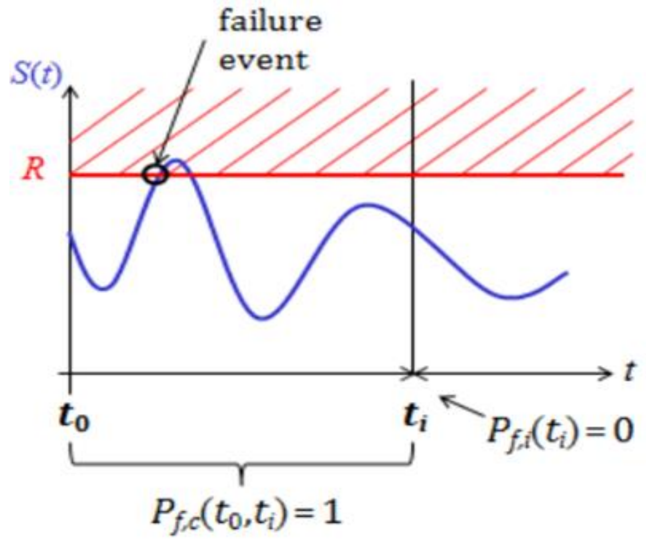
- n_X Random Variables(RV)
- $\boldsymbol{Y}(t)$ stochastic process represented by n_y RV
- May also explicitly depends on t

Objective:

▶ Calculate cumulative failure probability

$$P_{f,c}(0,t_l) = P(\exists t \in [0,t_l], G(\mathbb{X},t) < 0)$$

Challenge: potentially very expensive

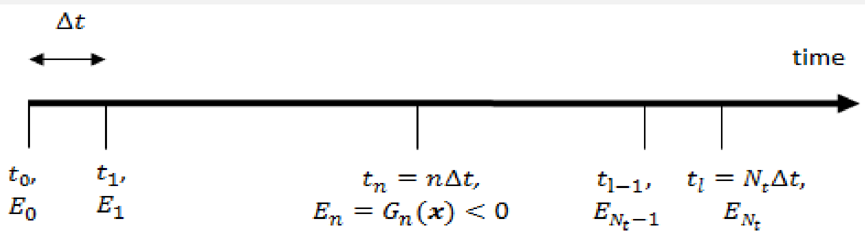


(Melchers 1999)

Time-dependent reliability analysis

One important step:

► Discretizing the time and converting the TVR into a static problem



$$t_n = n\Delta t, \; n = 0, \dots, N$$

$$G_n(X) = G(X, t_n)$$

$$E_n = \{G_n(X) \le 0\}$$

$$P_{f,c}(t_0, t_l) \approx P\left(\cup_{n=0}^{N_t} G_n(X) \le 0\right) = P\left(\cup_{n=0}^{N_t} E_n\right)$$

$\Rightarrow$ *Serial system* with p components

$$P_{sys} = P\left(\cup_{i=1}^{p} G_i(X) \le 0\right) = P\left(\cup_{i=0}^{p} E_i\right)$$

System reliability methods can be used for TVR

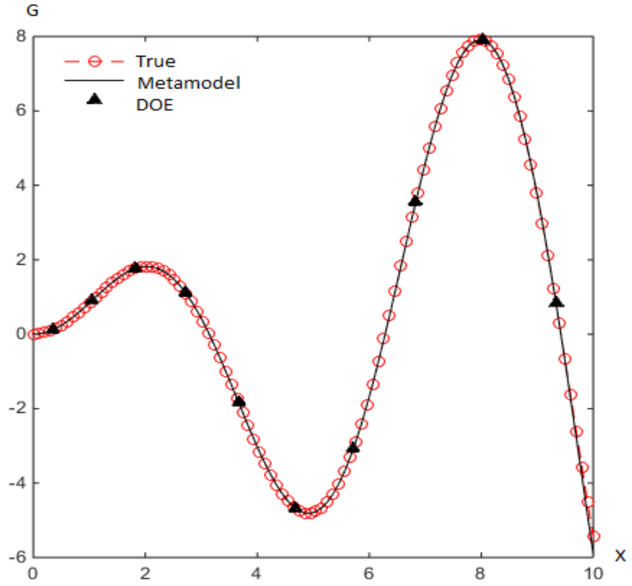
Meta-models

- Metamodels emulate behavior of real model

$$G: \mathbf{X} \in \mathcal{D}_{\mathbf{X}} \subset \mathbb{R}^{d} \rightarrow Y = G(\mathbf{X}) \in \mathbb{R}$$
$$G \sim \hat{G}$$

Advantages:
- Cheaper than original model
- DOE is sufficient to construct the model

DOE: a limited set of samples for which *G* is run



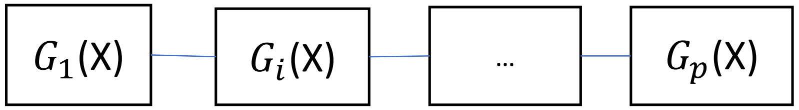
Number of calls to *G*: $N_{call} = N_{DOE} + N_{enrich}$

Different types: Kriging, Polynomial chaos expansion, Quadratic response surface…

(Hawchar et al. 2017)

System reliability: AK-SYS

► **AK-SYS**: Kriging-based system reliability method using an active learning process



$$P_{sys} = P\left(\bigcup_{i=1}^{p} G_i(X) \leq 0\right) = P\left(\bigcup_{i=0}^{p} E_i\right)$$

- Same approach as MCS is used to calculate the failure probability
- Performance functions are replaced by Kriging meta-models ($\hat{G}_i$)
- A composite criterion learning process is used to enrich the meta-models

➢ **Advantages**:

- Efficient: no optimization involved for the enrichment process
- General: no assumption on the limit state

(Fauriat and Gayton, 2013)

System reliability: AK-SYS

- **Composite criterion learning function:**

$$\mathrm{U_s}\left(\mathrm{X^{(i)}}\right) = \frac{|\widehat{\mathrm{G}}_\mathrm{s}(\mathrm{X^{(i)}})|}{\sigma_{\widehat{\mathrm{G}}_\mathrm{s}(\mathrm{X^{(i)}})}}$$

- For a given $\mathrm{X^{(i)}}, \mathrm{i} = 1, \ldots, N_{MCS}$: $\widehat{\mathrm{G}}_\mathrm{s}(\mathrm{X^{(i)}}) = \min\left(\widehat{\mathrm{G}}_\mathrm{j}\left(\mathrm{X^{(i)}}\right)\right), \mathrm{j} = 1, \ldots, \mathrm{p}$

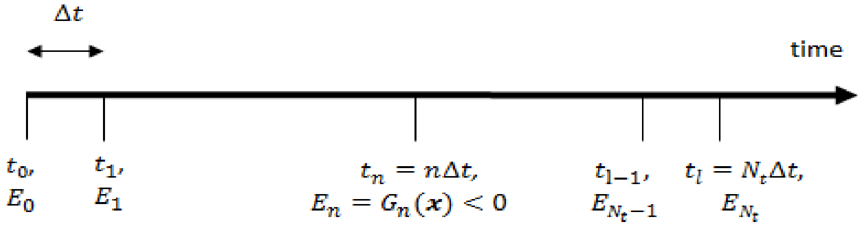
- $\mathrm{X^{(i)}}$ that minimizes $\mathrm{U_s}$ will be used to enrich the DOE
- The learning process continues until $\min\left(\mathrm{U_s}\right) > 2$
- Failure probability is calculated in the last step

$$\hat{P}_f = \frac{\hat{N}_{fail}}{N_{MCS}}$$

(Echard et al. 2011)

Proposed Methodology: AK-SYS-T

► We propose a new time-dependent reliability method by using the similarity between system reliability and time-dependent reliability



Time-dependent problem:

$$P_{f,c}(t_0, t_l) \approx P\left(\bigcup_{n=0}^{N_t} G_n(X) \le 0\right) = P\left(\bigcup_{n=0}^{N_t} E_n\right)$$

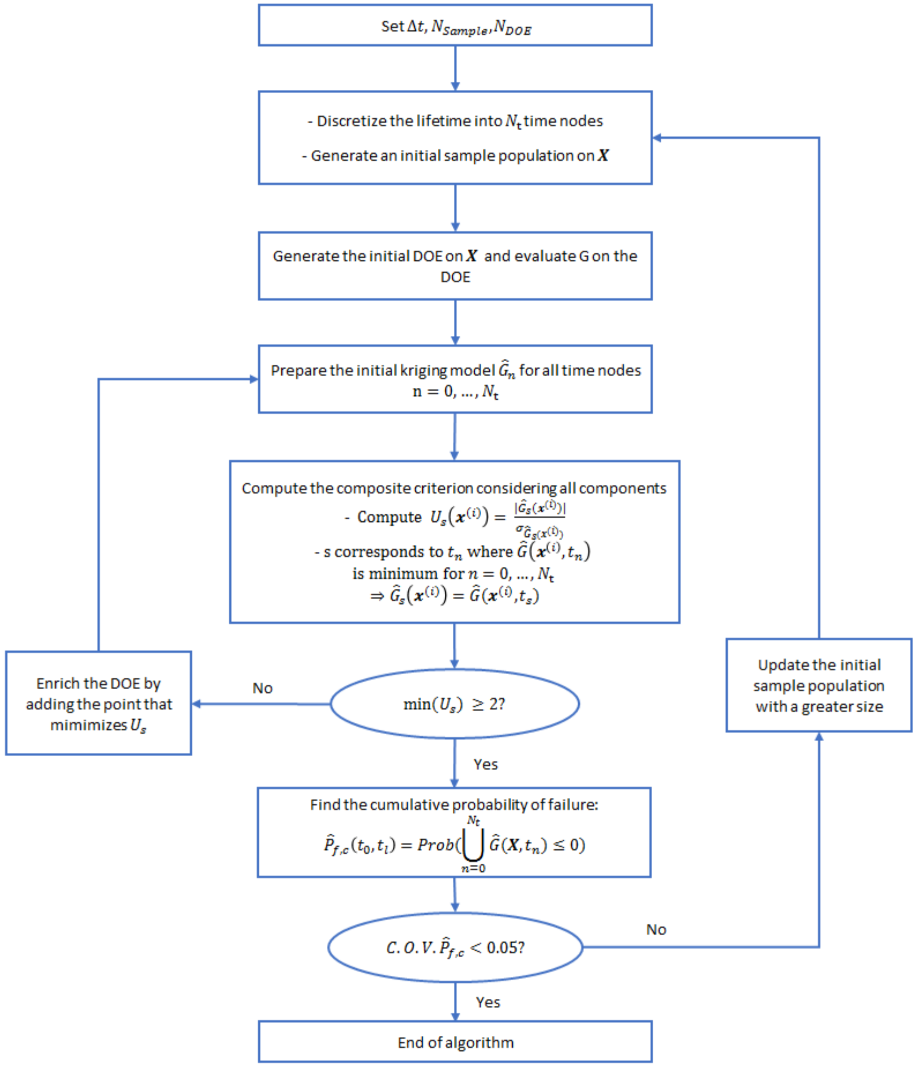
Serial system problem:

$$P_{sys} = P\left(\bigcup_{i=1}^{p} G_i(X) \le 0\right) = P\left(\bigcup_{i=0}^{p} E_i\right)$$

Proposed Methodology: AK-SYS-T

▶ **New methodology: AK-SYS-T**

- Discretizing the time interval, convert the problem into time-invariant
- Decomposition of stochastic processes if there is any
- Prepare initial Kriging metamodels for each time node
- Employing the composite criterion learning process to update the Kriging meta-models
- Computing the probability of failure



validation

▶ Validation:

➢ Comparison between AK-SYS-T and MCS

$$I\left(X^{(i)}\right) = \begin{cases} 1 & if \quad \widehat{G}\left(X^{(i)}\right) \times G\left(X^{(i)}\right) < 0 \\ 0 & if \quad \widehat{G}\left(X^{(i)}\right) \times G\left(X^{(i)}\right) \geq 0 \end{cases}$$

$$N_{misclass} = \sum_{i=1}^{N_{MCS}} I\left(X^{(i)}\right)$$

validation

Case study #1: Performance function with one RV

$$G(X,t) = \frac{1}{X^2+4}\sin(2.5X)\,cos(t+0.4)^2$$

$$X\sim N(10,1)$$

$$P_{f,c}(1,2.5) = Prob(\exists\,\tau \in [1-2.5],\ 0.014 - G(X,t) \leq 0)$$

Case 1:

Population size: 500000

DOE: 10 LHS

Same seed for MCS and AK-SYS-T

- Exact classification
- Number of time nodes is important

Time Nodes	P_MCS	P_AK-SYS-T	Misclass	N_calls
5	0.002048	0.002048	0	10+9
10	0.002046	0.002046	0	10+10
20	0.010068	0.010068	0	10+13
30	0.011448	0.011448	0	10+13
50	0.011588	0.011588	0	10+11

(Z. Hu and X. DU 2015)

validation

Case study #2: Performance function with 10 RV

$$G(\boldsymbol{X},\boldsymbol{Y}(t),t) = -\left(\frac{F(t)L}{4} + \frac{\rho_{st} a_0 b_0 L^2}{8}\right) + \frac{(a_0-2kt)(b_0-2kt)^2 \sigma_u}{4}$$

$$F(t) = 6500 + \sum_{i=1}^{7} \xi_i \left(\sum_{j=1}^{7}\left(a_{ij} \sin\left(b_{ij} t + c_{ij}\right)\right)\right)$$

$$P_{f,c}(0,35) = Prob(\exists\, \tau \in [0,35],\ G(X,t) \leq 0)$$

$$k = 5 \times 10^{-5}\ \frac{m}{year}$$

$$\rho_{st} = 7.85 \times 10^{4}$$

$$L = 5m$$

Variable	Mean	Standard deviation	Distribution
σ_u (Pa)	2.4×10^8	2.4×10^7	Normal
a_0 (m)	0.2	0.01	Normal
b_0 (m)	0.04	4×10^{-3}	Normal
ξ_1	0	100	Normal
ξ_2	0	50	Normal
ξ_3	0	98	Normal
ξ_4	0	121	Normal
ξ_5	0	227	Normal
ξ_6	0	98	Normal
ξ_7	0	121	Normal

(Z. Hu and X. DU 2015)

validation

Case 2:

Population size: 50000

DOE: 50 MCS

Same seed for MCS and AK-SYS-T

- There is some miss-classification
- The method is still very accurate

$$Error = \frac{|P-\tilde{P}|}{P} \times 100 = 4 \times 10^{-4}$$

Time Nodes	P_MCS	P_AK-SYS-T	Misclass	N_calls
5	0.04416	0.04418	7	50+23
10	0.044065	0.044025	12	50+29
20	0.044163	0.044063	17	50+27
50	0.044203	0.044243	14	50+ 27

Conclusion

▶ Conclusion and outcomes

- A new time-dependent reliability method has been developed
- This method borrows its efficiency and generality from AK-SYS

Publications

➢ Submitted:

- Time-variant reliability analysis based on AK-SYS, 13th ICASP, Seoul, South Korea, May 26-30, 2019

➢ To be submitted:

- AK-SYS-T: A new time-dependent reliability method based on Kriging meta-modelling, Journal of Reliability Engineering and System Safety

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