Multi-branch multi-state modeling for prognosis of systems under multiple deterioration mode

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Context

FP7 European project: "Sustainable Predictive Maintenance for manufacturing Equipment"

Purpose: Development of new tools for predictive maintenance to improve productivity, reduce machine downtimes and increase energy efficiency.

⇒ Application case: Paper machine

⇒ Main objectives:

✓ Deterioration modeling
✓ Remaining Useful Life (RUL) estimation
Deterioration modeling and RUL estimation

Deterioration modeling

- Synthesize health indicators from condition monitoring data
- Model its temporal evolution, e.g. by stochastic processes
- Assess component’s current health

Remaining Useful Life (RUL)

- The time left before the failure
- Prediction uncertainties management

=> RUL: Conditional random variable

Two main methods

- Continuous states modeling
- Discrete or multi-state modeling
Hidden Markov Model

- Two-layer stochastic process
  - States: Hidden ~ Markov chain (propriété Markovienne)
  - Observations: emitted by states through some probabilistic links

Deterioration modeling

- States: System’s health
- Observations: CM signals, high-level extracted features, etc.
- Left-right topology: irreversible deterioration
HMM modeling problems

- Model only one deterioration mechanism at one time
  Multiple deterioration modes could **co-exist in competition** in practice

Multi-branch Hidden Markov Model (MB-HMM)

- Markovian property: State sojourn time $\sim$ geometric or exponential distribution
  Can be extended to arbitrary distribution

Multi-branch Hidden semi-Markov Model (MB-HsMM)
Outline

- **Multi-branch multi-state model**
  - Deterioration modeling
  - Remaining Useful Life estimation

- **Numerical results**
  - Simulated degradation data
  - A case study

- **Conclusion & Perspectives**
Multi-branch modeling

- Several branches with a common initial (normal) state
- Each branch ~ one deterioration mode
- Branch probability $\pi_k$

$$\sum_{k=1}^{M} \pi_k = 1$$

- Observations are continuous ~ Gaussian distributed
- **Semi-Markov**: State sojourn times ~ Gaussian distributed
- Corresponding states in different branches could emit the same observations
Multi-branch framework for diagnosis and prognosis

Two-phase implementation: offline & online
Off-line phase

Model training

• Train data: High-level features extracted from condition monitoring data (RMS, kurtosis, crest factor, etc.)

• Data classification => M groups => train M branches separately

• Parameters estimation:
  – HMM: Baum-Welch algorithm
  – HsMM: Adoption of the Forward-Backward procedure [Yu06]

• *A priori* probabilities calculation

\[
\pi_k = P(\lambda_k) = \frac{K_k}{\sum_{k=1}^{M} K_k}
\]

\(K_k\): Number of training sequences corresponding to the mode \(k\)

On-line phase

Diagnosis

- Mode detection: \[ \hat{k} = \arg \max_k P(\lambda_k | O) \]

- Health-state assessment: Viterbi algorithm
  - Determine of the "best" state sequence: \[ Q^* = \arg \max_{Q_k} P(O, Q_k | \lambda_k) \]
  - Consider the last state as the actual state
RUL estimation

One branch (HMM case)

- Suppose that the system is following the mode $k$

- RUL: number of transition steps to reach \textit{for the 1st time} the failure state

\[
RUL_i^{(l)} = P(RUL = l \mid q_t = S_i) = P(q_{t+l} = S_N, q_{t+l-1} \neq S_N, \ldots, q_{t+1} \neq S_N \mid q_t = S_i)
\]

- Left-right HMM: Given the current state, the system can either stay in the same state or jump to the next one

$\Rightarrow \text{Recursive computation}$
RUL estimation (cont.)

One branch (HMM case)

- At state $S_{N-1}$:
  \[
  RUL_{N-1}^{(1)} = a_{(N-1)N}
  \]
  
  \[
  RUL_{N-1}^{(l)} = a_{(N-1)(N-1)} RUL_{N-1}^{(l-1)}
  \]

- At state $S_{N-2}$:
  \[
  RUL_{N-2}^{(1)} = a_{(N-2)N} = 0
  \]
  
  \[
  RUL_{N-2}^{(l)} = a_{(N-2)(N-2)} RUL_{N-2}^{(l-1)} + a_{(N-2)(N-1)} RUL_{N-1}^{(l-1)}
  \]

- At state $S_{i}$:
  \[
  RUL_{i}^{(1)} = a_{iN}
  \]
  
  \[
  RUL_{i}^{(l)} = a_{ii} RUL_{i}^{(l-1)} + a_{i(i+1)} RUL_{i+1}^{(l-1)}
  \]
RUL estimation (cont.)

One branch (HsMM case)

- Strictly left-right model:
  \[
  RUL_{t_i}^t = D_i^t + \sum_{j=i+1}^{N} D_j
  \]
  - \( D_j \): Sojourn time in states \( j \)
  - \( D_i^t = D_i - \bar{D}_i \mid D_i > \bar{D}_i \sim \text{truncated Normal distribution} \)
  - \( \sum_{j=i+1}^{N} D_j \sim \text{Normal distribution} \)

Bayesian Model Averaging

- Take into account model uncertainty:
  \[
  P(\text{RUL} \mid O) = \sum_{k=1}^{M} P(\text{RUL} \mid \lambda_k, O) P(\lambda_k \mid O)
  \]
Numerical examples

Simulated deterioration data

- **Fatigue Crack Growth (FCG)** model to represent the evolution of a crack depth

\[
x_t^i = x_{t-1}^i + e^{w_i} C \left( \beta e^{\gamma_e} \sqrt{x_{t-1}^i} \right)^n \Delta t
\]

- Observation model: \( y_t^i = x_t^i + \xi_t^i \)

- Multi-mode: Two propagation rates
  - Crack depth is proportional with \( \gamma_e \)
  
  \[
  \gamma_e = \begin{bmatrix} 0.005 & 0.75 \end{bmatrix}^T
  \]
  
  \[
  C = 0.005, \quad n = 1.3, \quad \sigma_w = 1.7
  \]
  
  \[
  \sigma_{\xi}^2 = 10, \quad L = 100, \quad \pi_1 = \pi_2 = 0.5
  \]
Numerical examples

Test data

- Failure time: $T_f = 682h$

RUL estimation

- Online RUL estimation
  - Time replacement: 50h
Numerical examples

Multi-branch model vs. Average model

- **Aims:** Assess advantages of the proposed multi-branch models in case of multi-modes with different mode “distance”
- **Mode “distance”:** proportional to $\gamma_e$
- **Criterion:** Root mean squared error (RMSE)

![Diagram of multi-branch model and average model](image.png)

**Graph showing comparison between AVG-HSMM and MB-HSMM models**
Case study:

PHM08 competition

- C-MAPSS: Modeling a large realistic commercial turbofan engine
- 2 data set for training and test
- 218 identical and independent units
- Objective:
  - Construct a prognostic method basing on training data set
  - Use it to estimate the RUL of each unit in test data set
- Evaluation criterion:
  \[ S = \sum_{i=1}^{218} S_i \]
  Where \( S_i \) is penalty score for unit \( i \):
  \[ S_i = \begin{cases} e^{-d_i/13} - 1, & d_i \leq 0 \\ e^{d_i/10} - 1, & d_i > 0 \end{cases} \]
  where \( d_i = RUL_{\text{est}} - RUL_{\text{real}} \)

*C-MAPSS: Commercial Modular Aero-Propulsion System Simulation*
Case study (cont.)

Application of the MB-HsMM

• Health indicator: From [Le Son et al. 2012]*
• Number of states determination
  – BIC criterion: N = 8 (fix M = 4)
• Number of branches determination:
  – K-fold cross validation with K = 10, N =8
    => $M_{opt} = 4$
• RUL estimation result

<table>
<thead>
<tr>
<th>Method</th>
<th>Score</th>
<th>RSE</th>
<th>MSE</th>
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<tr>
<td>1-branch HsMM</td>
<td>12246</td>
<td>502</td>
<td>1157</td>
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<td>2-branch HsMM</td>
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<td>3-branch HsMM</td>
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<td>4-branch HsMM</td>
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<td>684</td>
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<tr>
<td>Method in Le Son et al. (2012a)</td>
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<td>423</td>
<td>823</td>
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<tr>
<td>Method in Le Son et al. (2012b)</td>
<td>4107</td>
<td>434</td>
<td>864</td>
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Conclusion & Perspectives

Conclusion

• Proposition of Multi-branch multi-state model to deal with:
  – Co-existing deterioration mechanisms
• Proposition of an multi-branch based framework for diagnosis & prognosis
  – Detection of actual deterioration mode
  – Assessment of current health status
  – Estimation of the RUL
• Evaluate the efficiency of the proposed models through numerical studies

Perspectives

• Development of multi-branch model for continuous states case
• Allowing transitions between different modes
Thank you for your attention!

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